

Assignment 1

1. Approximately solve the boundary value problem

$$\begin{cases} e^{2y}u_{xx} - u_{yy} + u_y = 8e^{2y} \\ u(x, 0) = 0 \\ u_y(x, 0) = x^2 \end{cases}$$

in a neighbourhood of the x -axis by finding the first few terms of a formal Taylor series solution $y \mapsto u(x, y) = \sum_{k=0}^{\infty} c_k(x)y^k$ centred on the x -axis. Plot the first few partial sums. (Will this actually work?)

2. By expressing $\phi(r)$ as a formal power series (and, if you like, making the substitution $\rho = \lambda r$), show that there is only one *negative* value $\mu = -\lambda^2$ for which the problem

$$\begin{cases} -\left(\phi_{rr} + \frac{1}{r}\phi_r\right) = \mu\phi \\ \phi_r(1) = \phi(1) \end{cases}$$

admits a non-trivial solution. Denote this value by $\mu_0 = -\lambda_0^2$ and let ϕ_0 be the solution satisfying $\phi_0(0) = 1$.

3. Consider the problem

$$\begin{cases} (\partial_t - \Delta)u = 0 & \text{in } B^2 \times (0, \infty) \\ D_{\nu(x)}u(x, t) = u(x, t) & \text{at } S^1 \\ u(x, 0) = \phi_0(|x|), \end{cases}$$

where B^2 is the unit ball in $\mathbb{R}^2$ (centred at the origin) and $\nu(x) = x$ is the outward pointing unit normal to $S^1 = \partial B^2$. Find a (in fact, the) solution using separation of variables.

Assignment Due: Friday, 7th August by 17:00